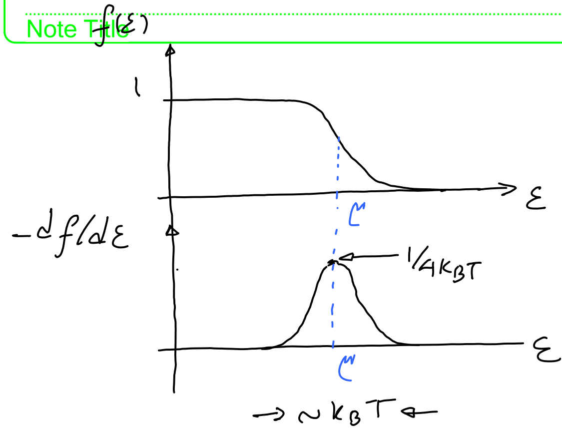
Session 4

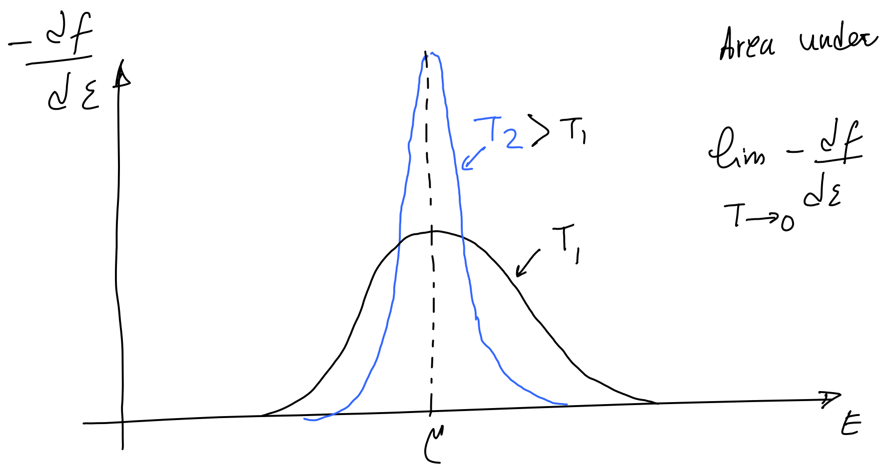


$$f(\varepsilon) = \frac{1}{1+e^{\frac{\varepsilon-\mu}{k_BT}}} \longrightarrow -\frac{\partial f}{\partial \varepsilon} = \frac{1}{k_BT}\,\frac{e^{\overbrace{\frac{\varepsilon-\mu}{k_BT}}^{\equiv x}}}{\left(1+e^{\frac{\varepsilon-\mu}{k_BT}}\right)^2}$$

$$= \frac{1}{k_BT}\,\frac{e^x}{(1+e^x)^2} = \frac{1}{k_BT}\,\frac{\cancel{e^x}}{\cancel{e^x}\underbrace{\left(e^{-x/2}+e^{x/2}\right)^2}_{\min = 2}}$$

$$\Rightarrow \operatorname{Max}\left(-\frac{\partial f}{\partial \varepsilon}\right) = \frac{1}{4k_BT}$$

↳ The area is almost constant.
So at zero kelven, it is a delta function



Area under curve is 1.

$$\lim_{T\to 0} -\frac{df}{d\varepsilon} = \delta(\varepsilon-\mu)$$

Conductance $G = \frac{I}{V} = \frac{2e^2}{h}\,\frac{\gamma_1\gamma_2}{\gamma_1+\gamma_2}\left[-\frac{df}{d\varepsilon}\right]$

It seems there is no limit on G.

However, as we will see, this is <u>not</u> true.

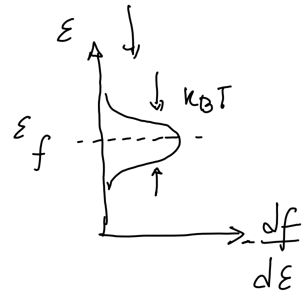
The maximum conductance for one energy level

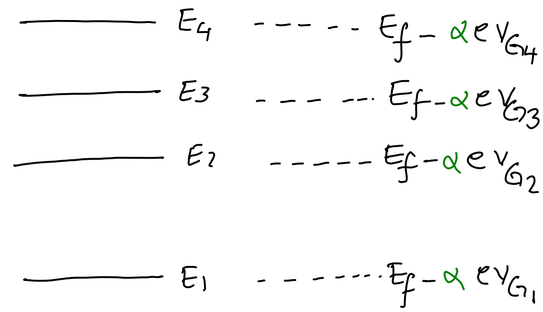
is: $\frac{2e^2}{h} \simeq 77.4\ \mu S \simeq \frac{1}{12.9\ k\Omega}$

or the minimum resistance for one energy level

is $12.9\ k\Omega$.

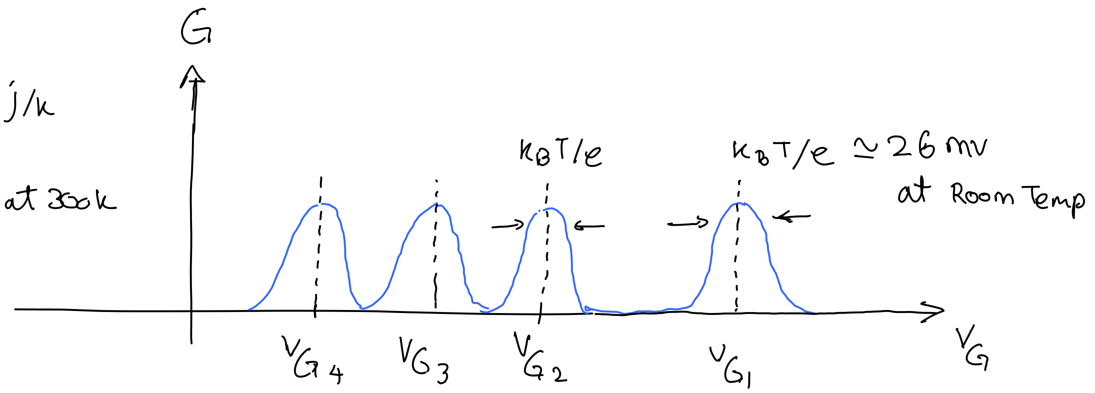
$$I = V \frac{2e^2}{\hbar} \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2} \left(-\frac{df}{d\varepsilon}\right) \quad \text{(Equ 1)}$$



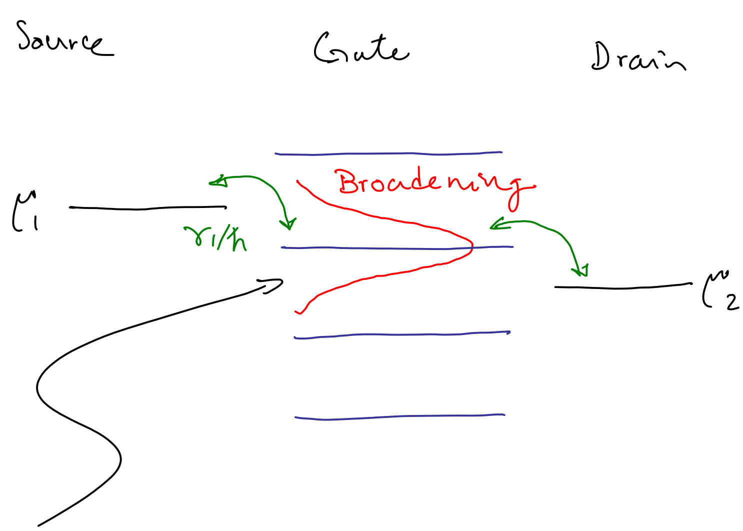


$k_B = 1.38 \times 10^{-23}$ J/K

$k_B T \cong 26$ meV at 300K



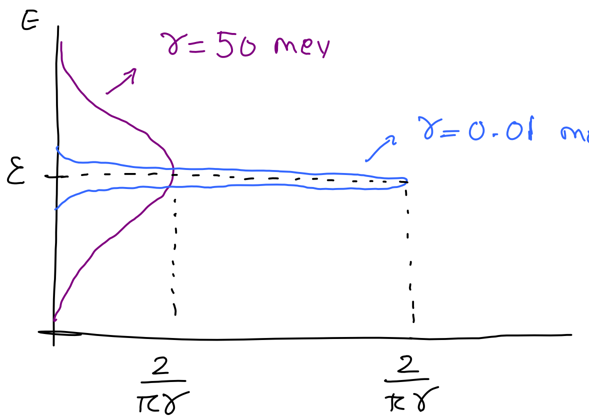
From Equ 1, it seems G can increase indefenitly with respect to the ratio $\frac{\gamma_1\gamma_2}{\gamma_1+\gamma_2}\left(-\frac{\partial f}{\partial \varepsilon}\right)_{max} = \frac{\gamma_1\gamma_2}{\gamma_1+\gamma_2}\frac{1}{4k_BT}$. However this is not true. The reason is that when the channele is connected to the contacts, they are broadened.



Energy levels lose descreteness and a broadened continous DOS is formed:

$$D(E) = \frac{\gamma/2\pi}{(E-\varepsilon)^2 + (\gamma/2)^2} \qquad \gamma = \gamma_1 + \gamma_2$$

Note: DOS can have other shapes. Above we assumed a simple Lorentzian function centered around energy ε.



$$\int_{-\infty}^{\infty} D(E)\, dE = 1$$

if $\gamma \to 0$, $D(E) \to \delta(E-\varepsilon)$

Reminder for F.T.:

$$F(\omega) = \int_{-\infty}^{\infty} e^{-i\omega t} f(t)\, dt$$

$$f(t) = \int_{-\infty}^{\infty} e^{i\omega t} F(\omega)\, d\omega$$

$$e^{-a|t|} \longleftrightarrow \sqrt{\frac{2}{\pi}} \frac{a}{a^2+\omega^2}$$

$$e^{iat} f(t) \longleftrightarrow F(\omega - a)$$

If we fourier Transform $D(E)$:

$$D(E) = \frac{\gamma/2\pi}{(E-\varepsilon)^2+(\frac{\gamma}{2})^2} \longleftrightarrow d(t) = ?$$

$$\frac{\gamma/2\pi}{(E-\varepsilon)^2+(\frac{\gamma}{2})^2} = \frac{\gamma/2\pi}{(\hbar\omega-\varepsilon)^2+(\frac{\gamma}{2})^2} \equiv \frac{\gamma}{2\pi\hbar^2}\,\frac{1}{(\omega-\frac{\varepsilon}{\hbar})^2+(\frac{\gamma}{2})^2}$$

$$\frac{1}{\omega^2+(\frac{\gamma}{2\hbar})^2} \longleftrightarrow \frac{2\hbar}{\gamma}\sqrt{\frac{2}{\pi}}\, e^{-\frac{\gamma}{2\hbar}|t|}$$

$$\frac{\gamma/2\pi}{(E-\varepsilon)^2+(\frac{\gamma}{2})^2} = \frac{\gamma}{2\pi\hbar^2}\,\frac{1}{(\omega-\frac{\varepsilon}{\hbar})^2+(\frac{\gamma}{2\hbar})^2} \longleftrightarrow \frac{\cancel{\gamma}}{2\pi\hbar^{\cancel{2}}}\,\frac{2\cancel{\hbar}}{\cancel{\gamma}}\sqrt{\frac{2}{\pi}}\, e^{\frac{i\varepsilon t}{\hbar}}\, e^{-\frac{\gamma}{2\hbar}|t|}$$

$$\Rightarrow d(t) = \frac{2^{1/2}}{\pi^{3/2}\hbar}\, e^{i\frac{\varepsilon}{\hbar}t}\, e^{-\frac{\gamma}{2\hbar}|t|} = \frac{2^{1/2}}{\pi^{3/2}\hbar}\, e^{i\frac{\varepsilon}{\hbar}t}\, e^{-\frac{|t|}{2\tau}}$$

$\tau \equiv \frac{\hbar}{\gamma}$ can be viewed as the lifetime of the particle.

$$\boxed{\text{Broadening in Energy}} \xleftrightarrow{\text{Fourier Trans.}} \boxed{\text{Life time in Time domain}}$$

So the current by having a DOS, $D(E)$ is:

$$I = V\int_{-\infty}^{\infty} dE\, D(E)\, \frac{2e^2}{\hbar}\,\frac{\gamma_1\gamma_2}{\gamma_1+\gamma_2}\underbrace{\left(-\frac{\partial f}{\partial E}\right)}_{\simeq\, \delta(E-E_f) \text{ at low } T}$$

$$\simeq V\,\frac{2e^2}{\hbar}\,\frac{\gamma_1\gamma_2}{\gamma_1+\gamma_2}\, D(E_f)$$ $\Rightarrow$ Conductance depends on DOS at Fermi energy.

$$D(E_f)_{max} = \max\left(\frac{\gamma/2\pi}{(E_f-\varepsilon)^2+(\gamma/2)^2}\right) = \frac{\gamma/2\pi}{\gamma^2/4} = \frac{2}{\pi\gamma} = \frac{2}{\pi(\gamma_1+\gamma_2)}$$

For a given DOS, $\gamma_1+\gamma_2=\gamma=$ const.

$$G_{max} = \frac{2e^2}{\hbar}\,\frac{\gamma_1\gamma_2}{\gamma_1+\gamma_2}\,\frac{2}{\pi(\gamma_1+\gamma_2)} = \frac{e^2}{\pi\hbar}\,\underbrace{\frac{4\gamma_1\gamma_2}{(\gamma_1+\gamma_2)^2}}_{\text{Maximum is 1 that happens when } \gamma_1=\gamma_2}$$

$$G_{max} = \frac{e^2}{\pi\hbar} = \frac{2e^2}{h} \simeq \frac{1}{12.9\,k\Omega}$$

$(\hbar = h/2\pi)$

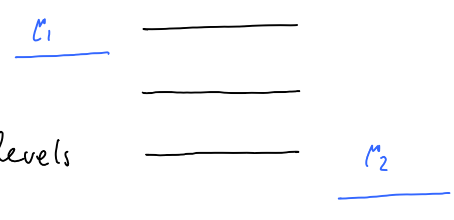
Quantum conductance

Now let's look at Ohm's Law

For short conductors, consider placing levels in series & in parallel:

$\boxed{\text{Parallel:}}$

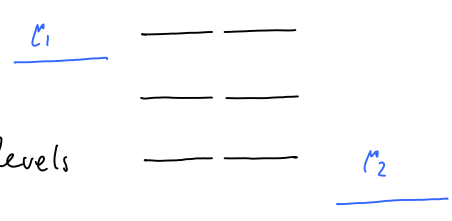
M levels



$\rightarrow G = \frac{2e^2}{\hbar} M$

$\boxed{\text{Series:}}$

M levels

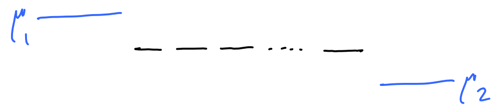


$\rightarrow G = ?$

Not that simple!
Electron transport is no longer ballistic as electrons now scatter before reaching the right contact.

If the electron mean free is L_0 & the total length of the channel L, we can write:

For series channels:

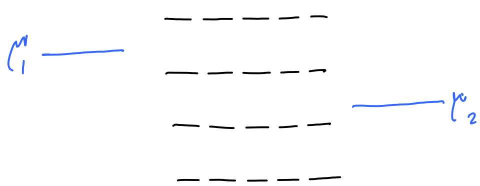


$$G = \frac{2e^2}{h} \frac{L_o}{L+L_o}$$

if $L_o \gg L \Rightarrow G = \frac{2e^2}{h}$

if $L_o \ll L \Rightarrow G = \frac{2e^2}{h}\frac{L_o}{L}$

For Parallel Series Combination:

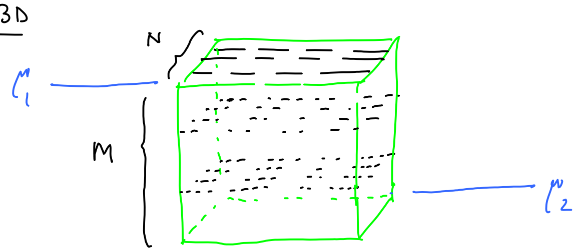


$$G = \frac{2e^2}{h} M \frac{L_o}{L+L_o}$$

For $L \gg L_o$: $G = \frac{2e^2}{h} \frac{L_o M}{L}$ = $\frac{2e^2}{h} \frac{\text{width}}{\text{length}}$ i.e. Ohm law

(width, length)

Or in 3D



$$G = \frac{2e^2}{h} MN \frac{L_o}{L+L_o}$$

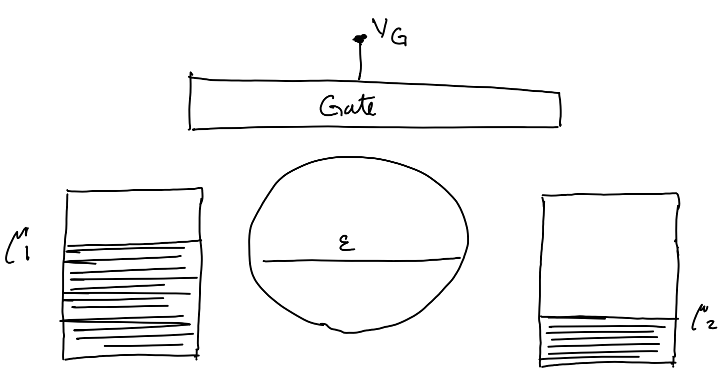
if $L \gg L_o \Rightarrow G \simeq \frac{2e^2}{h} L_o \frac{MN}{L} = \frac{2e^2}{h} \frac{\text{Area}}{\text{length}}$

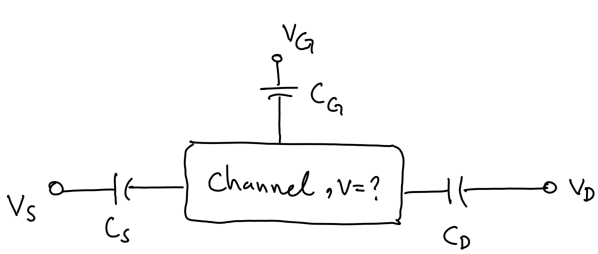
Ohm's law

Electrostatic of the channel

We learned that the gate voltage (V_G) moves the levels up or down. But how much?

We can model the terminals with capacitors connected to the channel:





what is the potential in channel:

from circuit: $V = \dfrac{C_S V_S + C_G V_G + C_D V_D}{\underbrace{C_S + C_G + C_D}_{\equiv C_T}}$

Set $V_S = 0$, i.e. source is grounded.

$$\Rightarrow \quad V = \frac{C_G}{C_T} V_G + \frac{C_D}{C_T} V_D \quad \text{or for energy:} \quad U_{ext} = \frac{C_G}{C_T}(-eV_G) + \frac{C_D}{C_T}(-eV_G)$$

Change in the energy levels in the channel due to external applied voltages.

Is this completely true?

No, this is true only if the number of electrons in the channel was not changing, i.e. if the channel was an insulator.

Increase in electrons → raises levels

Decrease → lowers levels

$$U = U_{ext} + U_{ee}$$

electron electrostatics

$$eV_{ee} = e\frac{\Delta q}{C_T} = e\frac{e\Delta N}{C_T} = \frac{e^2}{C_T}(N - N_0)$$

# of electrons in channel after applying external voltages

# of electrons at equilibrium

I $\quad U = U_{ext} + \dfrac{e^2}{C_T}(N - N_0) \quad \left(= \dfrac{C_G}{C_T}(-eV_G) + \dfrac{C_D}{C_T}(-eV_D) + \dfrac{e^2}{C_T}(N - N_0) \right)$

?

we need to know N, to find U. But we have already calculated N for single energy level:

$$N = \frac{\gamma_1 f_1 + \gamma_2 f_2}{\gamma_1 + \gamma_2} = \frac{\gamma_1 f(E-\mu_1) + \gamma_2 f(E-\mu_2)}{\gamma_1 + \gamma_2}$$

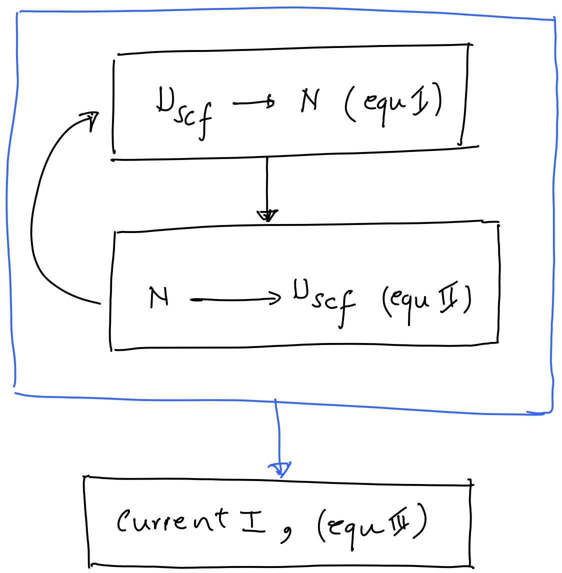
Here E was a single energy level. for a broadened energy level which is the more realistic case, we will have:

IV $$N = \int_{-\infty}^{\infty} dE\, D(E - U)\, \frac{\gamma_1 f_1(E) + \gamma_2 f_2(E)}{\gamma_1 + \gamma_2}$$

shift in channel energy

(III) $$I = \frac{e}{\hbar}\int_{-\infty}^{\infty} dE\, D(E)\, \frac{\gamma_1\gamma_2}{\gamma_1+\gamma_2}\left[f_1(E)-f_2(E)\right]$$

So U depends on N and N itself depends on U. We need to solve the two equations of (I) & (II) selfconsistently to find N & U. Therefore, we call U, the self consistent field, U_{scf}:



What is the effect on conductance vs. Gate voltage?

Suppose that the gate is very closely coupled to channel so C_G is dominant:

$$\frac{C_G}{C_T} \simeq 1 \qquad \frac{C_D}{C_T} \approx 0 \quad \rightarrow \quad U_{ext} \cong -eV_G$$

$$U_{scf} = U_{ext} + \frac{e^2}{C_T}(N-N_0) = -eV_G + \frac{e^2}{C_G}(N-N_0)$$

$$G = \underbrace{\int_{-\infty}^{\infty} dE\, D(E-U_{scf})}_{\text{Broadening}} \frac{2e^2}{\hbar}\frac{\gamma_1\gamma_2}{\gamma_1+\gamma_2}\underbrace{\left(-\frac{df}{dE}\right)}_{\text{Temperature effect, } k_B T}$$

U_{scf}: depends on N

$U_0 = \frac{e^2}{C_T}$ in $\frac{e^2}{C_T}(N-N_0)$